



Lattice Path Counting and Optimal Utilization of Multi-Channel Multi-Server Resources: Decorating the Math Magic Flower

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ABSTRACT

Inspired by a recent study of a Math Magic Circle [MMC], we undertake a similar study of a mathematical construct -- called Math Magic Flower [MMF]-- involving the first 81 natural numbers $\{1, 2, \dots, 81\}$ - each one being used exactly once. Of these numbers, only one viz., the number 41 acts as an isolate and it occupies a special position in the MMF. The geometry exhibits the number 41 holding a central position, better identified as one acting as the centre of six concentric circles. Along the circumferences of these circles are placed the natural numbers - each circle holding 10 of the numbers. As a matter of fact, the third and the fourth circles accommodate 20 numbers each. Thus we have the count $81 (= 1 + 10 + 10 + 20 + 20 + 10 + 10)$. The placement of the numbers is very special. There are intersecting straight lines with each line supporting exactly 3 of the 80 numbers and more over, the sum of the three numbers is the same ($=123$) for all lines. Our application of the magic flower deals with a navy pier operating from the central position towards outwardly navigation of ships. Or else, it could as well be in the opposite direction - from the farthest position on the sixth circle towards the centre touching each of the circles in between. There is a sense of Lattice path counting towards optimal utilization of Multi-Channel Multi-Server Resources.

Keywords: Navy pier; Loading/unloading zones; Cargo ship; Route, Optimal route

1. Introduction

Sometime back, one of the authors [bks] was introduced to a Math Magic Circle [MMC]. It was a fascinating mathematical construct and its statistical perspectives were nicely developed. Indeed the study was quite instructional and at the same time entertaining.

See Figure 1.

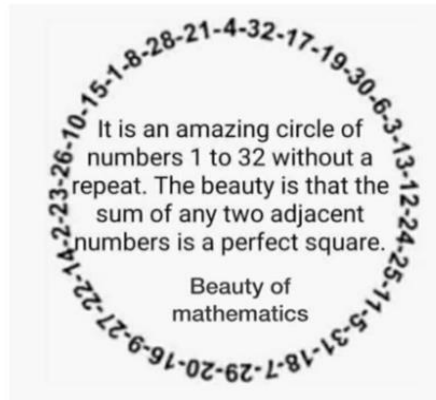


Figure 1. Math Magic Circle

This time we have come across a "Math Magic Flower" [MMF] shown below in Figure 2. In Section 2, we explain various features of the MMF. Next in Section 3, we develop an application-oriented scenario to utilize the MMF. Lattice path counting comes into play and we embark on multi-channel and multi-server resources. Statistical perspectives are brought into the data analysis. We conclude with mention of some open problems in Section 4.

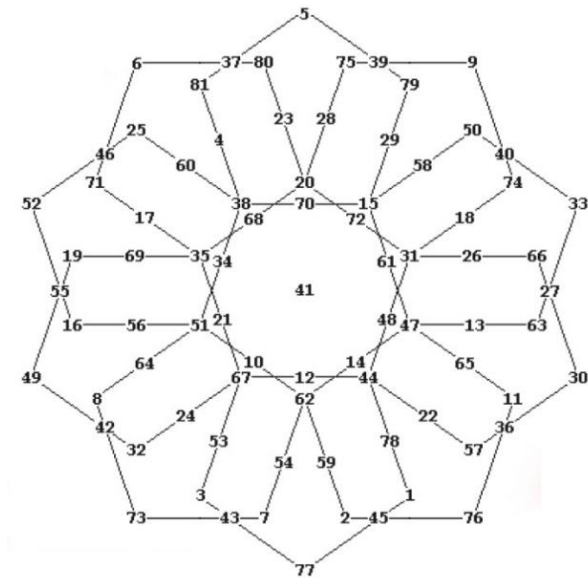
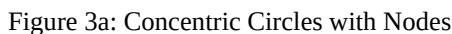


Figure 2. Math Magic Flower

We refer to Niwin *et. al.* (2026) for a statistical study of MMC. In this paper we will deal with the MMF and discuss its statistical perspectives with reference to a navy pier and its operation [dealing with cargo ships] in the background.

From a distance, the MMF indeed looks like a flower decorated with natural numbers from 1 to 81. A closer look shows that the number 41 is acting like an 'isolate'. Yes ! We also visualize circles --- concentric circles -- and there are six of them. See Figure 3.



Along the circumferences of each of the circles, we find placement of ten distinct integers. There is something more to it. As many as twenty integers are used to decorate each of the third and the fourth circles. There are perfectly decorated pentagons as well. Except for the integer 41, all others have landed on straight lines with two other numbers - all being distinct from each other. Interestingly, the sum total of these three numbers is the same for all lines.

To develop and discuss statistical feature for this MMF, we contemplate that this MMF exhibits a navy pier spread over a huge area and each number provides a dedicated location for providing service to cargo ships as and when they are brought to these stations. We also assume that the ten different locations along each circle provide same type of service but with varying duration and cost. The cost at any location is taken to be the corresponding number [in suitable units].

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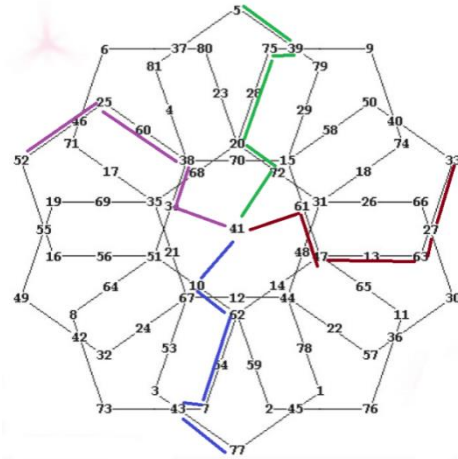


Figure 4. Descriptions of Four Different Routes

In Example 1, d0 refers to the origin in the whole process of activities of the cargo ships. d1, d2... refer to the positions of the concentric circles. We note that d3a and d3b refer to the two directions naturally originating from the second concentric circle and proceeding towards the third. In Figure 4 the route indicated by green is shown in the Example 1.

Example 1: d0(41)->d1,2(72)->d2,1(20)->d3,a;1 (28)->d4,a;1(75)->d5,1(39)->d6,1(5)

This example demonstrates the flow chart of a cargo ship originating from the centre d0 (41) and going upwards along the route as indicated. Thus it passes through d12(72) in the first concentric circle. Next it goes to d2,1(20), followed by d3,a;1 (28), next to d4,a;1(75) followed by d5,1(39) and finally reaches d6,1(5). During the whole process of movement of the ship from the origin to the extreme outermost circle, the cargo ship is supposed to receive adequate service at every place it is stopping. The quantity shown inside the bracket at every halt is the cost incurred for availing of the service. This is a fixed quantity irrespective of the route followed and independent of the volume of the work provided. In a very general sense the service cost has been simplified irrespective of the nature of the cargo ship.

We now indicate the total cost involved during the movement from d0 to d6 following the route shown in Example 1:

Total Cost = 280

In Figure 4, we had also traced three more routes indicated in different colours. The special roles of the concentric circles 3 and 4 are to be remembered while working on the path counting exercise.

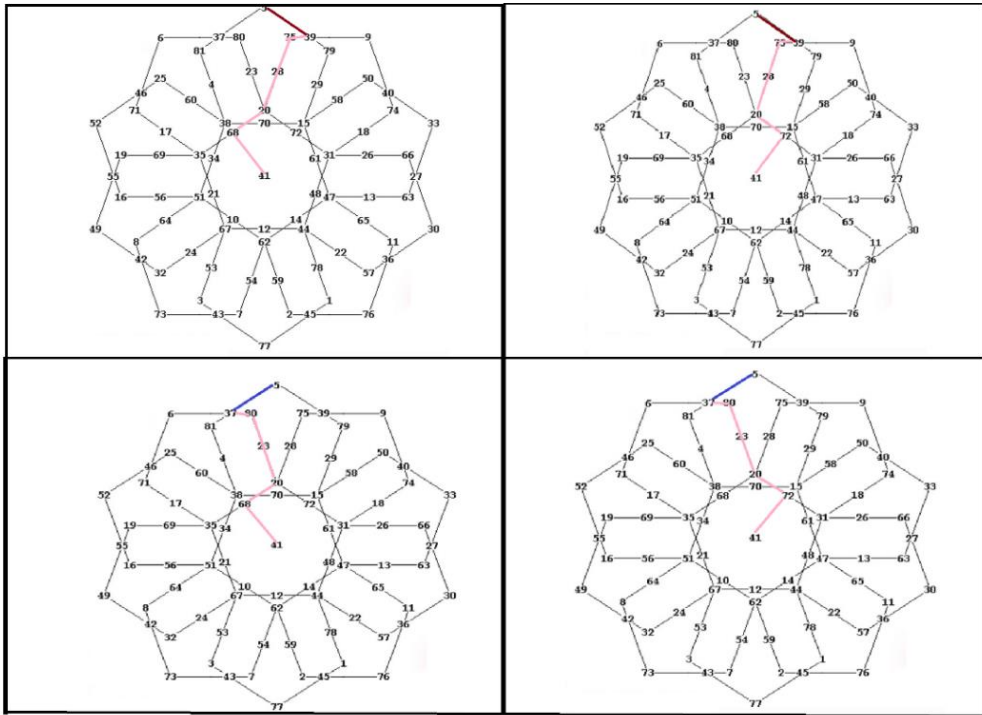
Red: d0(41)->d1,3(61)->d2,4(47)->d3,b;6(13)->d4,b;6(63)->d5,3(27)->d6,3(33)

Total Cost = 285

Blue: d0(41)->d1,7(10)->d2,6(62)->d3,a;11(54)->d4,a;11(7)->d5,6(43)->d6,6(77)

Total: 294

It is revealing to notice that every cargo ship, to start with, touches the first and second concentric circles before moving forward. For every movement of a cargo ship there is a pentagon which is involved as a centre for providing the necessary service. There are instances of routes passing through exactly one side of a pentagon. Here are four such routes. Routes passing through exactly one side of a pentagon:



Example 2:

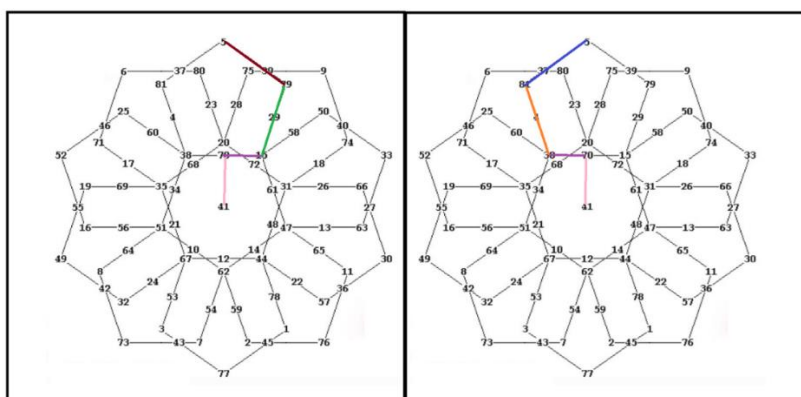
1. d0(41)->d1,2(72)-d2,1(20)->d3,a;1(28)->d4,a;1(75)->d5,1(39) red ->d6,1(5)
2. d0(41)->d1,10(68)->d2,1(20)->d3,b;20(23)-d4,b;20(80)->d5,10(37) blue->d6,1(5) red cum blue
3. d0(41)->d1,2(72)-d2,1(20)-> d3,b;20(23)-d4,b;20(80)->d5,10(37) blue->d6,1(5) red cum blue
4. d0(41)->d1,10(68)->d2,1(20)-> d3,a;1(28)->d4,a;1(75)->d5,1(39) red -d6,1(5) red cum blue

Passing through two sides of a pentagon is not possible.

For passing through three sides, we have many routes.

1. d0(41)->d1,1(70)-d2,2(15)->d3,b;2(29)->d4,b;2(79)->d5,1(39)->d6,1(5) red cum blue
2. d0(41)-d1,1(70)->d2,10(38)->d3,a;19(4)->d4,a;19(81)->d5,10(37)->d6,1(5) red cum blue

Example 3:



Below in Table 1 we display the routes followed by the cargo ships with reference to movements along different sides of a pentagon.

Table 1

SI No	Location: (outermost)	Path/Route	Location: (outermost)	Path/Route
1	5; external right	41,70,15,29,79,39,5	5; external left	41,70,38,4,81,37,5
		Total: 278		Total: 276
2	5; internal right	41,72,20,28,75,39,5	5; internal left	41,68,20,23,80,37,5
		Total:280		Total:274
3	9; external right	41,72,31,18,74,40,9	9; external left	41,72,20,28,75,39,9
		Total: 285		284
4	9; internal right	41,61,15,58,50,40,9	9; internal left	41,70,15,29,79,39,9
		Total:274		Total: 282
5	33; external right	41,61,47,13,63,27,33	33; external left	41,61,15,58,50,40,33
		Total: 285		Total: 298
6	33; internal right	41,48,31,26,66,27,33	33; internal left	41,72,31,18,74,40,33
		Total: 272		Total 309
7	30; external right	41,48,44,22,57,36,30	30; external left	41,72, 31,26,66,27,30,
		Total: 278		Total: 293
8	30; internal right	41,14,47,65,11,36,30	30; internal left	41,61,47,13,63,27,30
		Total: 244		Total: 282
9	76; external right	41,14,62,59,2,45,76	76; external left	41,14,47,65,11,36,76
		Total: 299		Total: 290
10	76; internal right	41,12,44,78,1,45,76	76; internal left	41,48,44,22,57,36,76
		Total: 297		Total: 324
11	77; external right	41,12,67,53,3,43,77	77; external left	41,12,44,78,1,45,77
		Total: 296		Total
12	77; internal right	41,10,62,54,7,43,77	77; internal left	41,14,62,59,2,45,77
		Total: 298		Total: 300
13	73; external right	41,10,51,64,8,42,73	73; external left	41,10,62,54,7,43,73
		Total: 289		Total: 290

14	73; internal right	41,21,67,24,32,43,73	73; internal left	41,12,67,53,3,43,73
		Total: 301		Total: 292
15	49; external right	41,21,35,69,19,44,49	49; external left	41,21,67,24,32,42,49
		Total: 278		Total: 276
16	49; internal right	41,34,51,56,16,55,49	49; internal left	41,10,51,64,8,42,49
		Total: 302		Total: 265
17	52; external right	41,34,38,60,25,46,52	52; external left	41,34,51,56,16,55,52
		Total: 296		Total: 305
18	52; internal right	41,68,35,17,71,46,52	52; internal left	41,21,35,69,19,55,52
		Total: 305		Total: 292
19	6; external right	41,68,20,23,80,37,6	6; external left	41,68,35,17,71,46,6
		Total: 275		Total: 284
20	6; internal right	41,70,38,4,81,37,6	6; internal left	41,34,38,60,25,46,6
		Total: 277		Total: 250

Remark 1: In the above table we have shown the paths touching or crossing at least one of the sides of the pentagon. There are an equal number of paths or routes going through the interior of the pentagon. We have not listed them here.

Remark 2: We carried out this exercise to communicate to the readers at large the niceties of the flower-shaped mathematical construct.

4. Other Considerations

Although under normal conditions the cargo ships will operate according to the designated routes, there are possibilities of mal functioning of some of the locations before/during/after providing a service to a cargo ship. Once this happens, it is likely that several ships will get involved and their planned services will be hampered. At that point of time, one has to work out alternate routes which are likely to be more time consuming or more costly or both.

Other possibility is some kind of disruption in the services which are caused by employee unrest, shortage of supply of necessary material etc. These unforeseen circumstances would have undesirable consequences in terms of time loss/budgetary limits/ unsatisfactory work/the like. Besides these issues, there could be several probability models governing the paths of the cargo ships whenever branching occurs.

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